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VOLATILITY VIA SOCIAL FLARING

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ABSTRACT:

A new explanation of kurtosis in asset price behavior is proposed involving flare attractors. Such attractors depend on chaotic fundamentals driving subsystems which trigger nonlinearly response functions each with a switching mechanism representing the changing of agents from stabilizing to destabilizing behavior. Heterogeneous agent types are shown by a set of these response functions that are interlinked. With a larger number of agent types system behavior resembles that of many financial markets. Such a model is consistent with newer approaches relying upon evolutionary learning mechanisms with heterogeneous agents as well as models depending on fractal characteristics.

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1. Introduction

Since Mandelbrot (1963), it has become an accepted stylized fact of asset markets that they exhibit considerable kurtosis, commonly identified as Aexcess volatility.≅ His initial hypothesis of asymptotically infinite variance fell by the wayside, but his approach inspired the development of ARCH analysis which has been widely applied to model volatility clustering in financial markets (Engle, 1982). However this approach has been shown to overpredict and underpredict outcomes in consistent ways (Brock, Lakonishok, and LeBaron, 1992). Mandelbrot (1997) in turn proposes the use of multifractals to model volatility in asset markets, although this approach seems to lack a theoretical economic foundation. Yet another variation on the original Mandelbrot approach is that of Loretan and Phillips (1994) who suggest the existence of asymptotically infinite fourth moments with finite variance as an explanation. Yet another approach has been to suggest the eruption of nonstationarities (de Lima, 1998), although the explanation of why large nonstationarities should erupt is not given, implicitly reflecting some kind of noisy deep shocks..

Another suggested approach is that asset markets may follow chaotic dynamics (Blank, 1991; Eldridge, Bernhardt, and Mulvey, 1993). However, this has been questioned by various researchers (Jaditz and Sayers, 1993; LeBaron, 1994). Curiously, although many thought this might explain the large stock market crash of October, 1987, the boundedness of chaotic dynamics actually suggests that this is not such a good method for modeling the kinds of extreme behaviors that generate the ubiquitous kurtosis of financial markets. An older argument that is more consistent with scattered events of very large changes involves the use of catastrophe theory (Zeeman, 1974; Gennotte and Leland, 1990; Rosser, 1997). But this approach involves numerous restrictive assumptions that leave many observers dissatisfied.

In a search for theoretical underpinnings many researchers emphasize the role of heterogeneous agents with destabilizing chartist traders interacting with stabilizing fundamentalist traders or others (DeLong, Shleifer, Summers, and Waldmann, 1990; Day and Huang, 1990; Ahmed, Koppl, Rosser, and White, 1997; Lux, 1998). A more recent development has been to emphasize learning behavior by such agents (Arthur, Holland, LeBaron, Palmer, and Tayler, 1997) with some models placing this in the context of social interactions between the agents that can shift discontinuously in the manner of phase transitions from statistical mechanics (Durlauf, 1993; Brock, 1993; Brock and Hommes, 1997).

This paper proposes a new approach to financial market volatility that draws on several of these elements. The basic idea is to assume that the external driver on yields and thus the determinant of fundamental expected present values is a chaotic dynamic. There are a variety of agent types each of whom is linked to a passively reacting social interactions model with threshold effects for each agent type for switching between destabilizing chartist trading and stabilizing fundamentalist trading..

It will be shown that for such a system each agent type can track a *flare attractor* (Rössler and Hartmann, 1995) that has been suggested as a possible model for solar flares and certain kinds of autocatalytic chemical reactions as well as for volatility of entrepreneurial outcomes (Hartmann and Rössler, 1998). Such attractors are generic examples of singular-continuous-nowhere-differentiable attractors (Rössler, Knudsen, Hudson, and Tsuda, 1995) that exhibit Ariddled basins≅ (Alexander, Yorke, You, and Kan, 1992) due to the intermingling of bounded and unbounded attracting basins, which are special cases of Milnor (1985) attractors. They also belong to the broader category of Kaplan-Yorke (1979) attractors that possess an absolutely smaller negative Lyapunov exponent for the reacting equation than the absolute value of the positive Lyapunov exponent for the forcing chaotic function. Hartmann and Rössler (1998) show that such systems are quite robust to a wide variety of specifications and also that they exhibit certain fractal characteristics as well. Thus, such models offer an attractive alternative that combines elements of several recent models that have been suggested for explaining the extreme volatility seen in financial markets.

A particularly interesting aspect of this involves considering a Asociety of flare attractors≅ in which a set of such attractors that vary slightly in their basic thresholds and initial conditions are coupled. Each of these can be viewed as representing an agent type that is reacting to the behavior of the other agents. In such a system with only one agent, the behavior appears to alternate too sharply between fairly simple chaotic behavior and dramatically kurtotic flaring episodes. But with a larger number of agents the behavior r of the system becomes more nuanced and complex and more resembles that seen in many asset markets, while still exhibiting kurtosis-generating flaring episodes. Such an approach seems to offer excellent possibilities for being incorporated into models in which agents learn and can move from agent type to agent type as with the Arthur, Holland, LeBaron, Palmer, and Tayler (1977) model.

2. A Model of the Fundamentals Process

Our model involves two stages, a process driving fundamentals and a set of reaction functions by the various agent types. We shall follow Day (1982) by presuming that the fundamentals driving process is a chaotic dynamic driven by a logistic function. Such a formulation can arise from a Cobb-Douglas technology in which the capital stock faces a productivity limit, possibly due to an environmental constraint as suggested by Day. The income streams of assets are seen as a deriving from a fixed percentage of this chaotic dynamic, however the actual income streams may be smoothed following the argument of Marsh and Merton (1986). Nevertheless, the agents will be reacting to the fundamentals driving process in their buying behavior, perceiving it as an information generating process with regard to future yields. Given its chaotic nature they may perceive it as simply a random process, although the results are not sensitive to whether or not they perceive the true nature of the underlying process. What is more significant is the nature of their reaction functions which will each switch between stabilizing fundamentalist behavior and destabilizing trend-chasing (chartist) behavior at different thresholds. Figure 1 displays a basic picture of how this model looks for a single agent type, with the chaotic attractor triggering the threshold switch at a critical value.

[insert Figure 1]

Day (1982) presents a modified Solow-type growth model where k is the capital-labor ratio, s is savings given by $s(k) = \alpha$, $f(k)$ is the production function, m is a saturation level of k due to pollution or some other Acapital-congestion≅ effect, and λ is the population growth rate. The capital-congestion effect can arise in urban systems without an environmental component. Thus

$$f(k) = Bk^{\beta}(m-k)^{\gamma}. \tag{1}$$

Maintaining the assumption of a constant savings rate this then implies a difference equation form for the growth of the capital-labor ratio given by

$$k_{t+1} = \alpha B k_t^{\beta}(m-k_t)^{\gamma}/(1+\lambda). \qquad (2)$$

This is a form of the logistic function whose chaotic behavior has been studied by May (1976) and others at length. In particular, for the special case of $\beta = \gamma = m = 1$ which can arise in the case of normalization of m, it is well known that k will behave chaotically if

$$3.57 \# \alpha B/(1+\lambda) \# 4. \qquad (3)$$

We emphasize that this is hardly the only growth model that can generate such chaotic dynamics, although it is a reasonably familiar one, with Rosser (2000) reviewing a variety of others. However, for our purposes in this paper we shall assume that the growth of k is given by such an equation where $\alpha B/(1+\lambda) = 3.99$ and the rest of the special case assumptions hold, that is

$$k_{t+1} = 3.99(1-k_t). \qquad (4)$$

In turn we shall assume that asset yields are seen as a smoothing of a fixed percentage of k as it chaotically evolves. This means that income distribution is not determined by marginal products of the production function as it is well known that for the Cobb-Douglas the shares going to respective factors do not change with factor ratios. Rather we shall assume that the rate of profit on capital is fixed reflecting a social bargaining process as argued by Cambridge capital theorists (Robinson, 1953-54), thus giving the amount of income going to capital as a fixed percentage of the capital-labor ratio.

3. Agent Reaction Functions

Although we shall not explicitly introduce a learning process in this model, we shall see how such a process might lead to an evolution of the system as more and more agent types develop, albeit with minor variations on each other. Our agent types will all contain conflicts between stabilizing behavior and destabilizing behavior that derives from the collective nature of agents= behaviors. Also, each agent=s response function will contain a switch point between overall stabilization and overall destabilization with this basic conflict in place in all cases. This basic pattern is depicted above in Figure 1 where the fundamental chaotic dynamic drives the agent above or below the switch point

Following Hartmann and Rössler (1998) we assume the general form of this agent reaction function for agent type I with I = 1 to n to be

$$b_{t+1}^{(I)} = b_t^{(I)} + b_t^{(I)}(a^{(I)} - k_t^{(I)}) - cb_t^{(I)2} + cs_t \qquad (5)$$

where $b_t^{(I)}$ is the asset demand by agent type I in time t with $0 < a < 1$ and $c > 0$. The final term s_t will reflect recent overall market demand and will thus be given by

$$s_{t+1} = b_t^{(1)} + b_t^{(2)} + \ldots + b_t^{(n)}. \qquad (6)$$

The first term in (5) provides an autoregressive component. The second is the switching term. The third provides a stabilizing component. The fourth is the destabilizing element coming from the trend-chasing or chartist aspect. However, it is the second term which determines which kind of behavior will be dominant. This second term will depend on the evolution of k_t. Although it is a simple linear equation, its role as a switching term arises from its ability to change sign. This means that the term a serves as a critical threshold such that if the value of $(a-k_t)$ changes sign then the qualitative behavior of the agent changes.

We shall allow n to go up to 18. Also we shall allow each agent type to face its own virtual capital-labor ratio, $k_t^{(I)}$, reflecting a specific portfolio it possesses, although the dynamics for all such ratios will be given by (4). Furthermore, we shall assume that $c = 10^{-3}$ and that the initial values for all the b=s will be 0.2. The initial values of the k=s will be given by $k^{(I)} = 0.010$, $k^{(2)} =$

0.011, . . ., $k^{(18)} = 0.028$. Finally, the a=s are given by $a^{(1)} = 0.565$, $a^{(2)} = 0.566$, . . ., $a^{(18)} = 0.582$.

4. Model Behavior

Simulations up to 1,000 iterations for these assumptions are shown for Figures 2-5, drawing on Hartmann and Rössler (1998, p. 155). Figure 2 shows the case for just one agent type given by the characteristics for (1) with no final term because there are no other agents to accumulate demands from.. Figure 3 shows the case for three agent types with characteristics for (1) through (3). Figure 4 shows the case for six agent types, (1) through (6), and figure 5 shows the case for all 18 agent types as given. Clearly as the number of types increases the system goes from exhibiting more scattered but more relatively dramatic flares or speculative outbursts to a more nuanced pattern that more resembles what we see in actual asset markets, while retaining the kurtotic aspect induced by the flaring. We note that this model simply adds the agent types to the system, although this should not alter the qualitative dynamics resulting.

[insert Figures 2-5]

We note that Figure 2 shows closer detail of the nature of the extreme flare produced in this case. Essentially what is involved is a very strong positive feedback that is however still ultimately limited in its explosiveness. These limits exhibit themselves in the fallbacks that occur on the way up as well as related spikes that occur on the way down after the flare reaches its maximum point. This reflects the pattern of speculative bubbles that do not simply go up and then down but show greater complexity of motion as they do so with smaller oscillations along the way.

To test whether this model can generate the kinds of qualitative dynamics seen in actual financial markets we consider the model of agents, the behavior of which is depicted in Figure 5. Figure 6 shows a histogram of the series generated by the 18 agent model depicted in Figure 5, and Table 1 presents certain associated statistics. We note in particular the measure for the value of kurtosis of 4.397952. The associated Jarque-Bera statistic indicates that the null hypothesis that this would equal the kurtosis value associated with a Gaussian normal distribution is to be rejected with a probability well in excess of 99 percent. This kurtosis value corresponds quite closely to those found for the one and two factor models of Treasury bills series in Tables 13.3 and 13.4 of Engle, Ng, and Rothschild (1990).

Finally, estimates by the authors find significant ARCH and GARCH effects as well, which is consistent with the behavior of most financial markets (available on request from the authors). Thus, this approach is capable of replicating some of the more unusual but ubiquitous characteristics of most financial time series that are not so easily explained or modeled, especially kurtosis.

[insert Figure 6 and Table 1].

5. Discussion

Although these results possess potential appeal for a number of reasons, we must recognize some caveats associated with them. An important one is the use of chaotic dynamics to model the underlying fundamentals process. As already noted there is much doubt as to whether any economic series are truly chaotic. Furthermore, even if they are there arises the question of whether or not agents can understand or mimic such dynamics, given the doubts that have been raised regarding the ability of agents to form rational expectations in situations in which there is sensitive dependence on initial conditions as is the case with chaotic dynamics (Grandmont, 1985).

With regard to the first question we shall not attempt to answer that as it has been much discussed elsewhere (Dechert, 1996). However, we note that if an alternative is some kind of purely stochastic exogenous process, then a chaotic dynamic can mimic that in interesting ways, although not perfectly. One difference is that a chaotic dynamic may well stay more bounded than will a purely stochastic process. Nevertheless the social flare attractor model under consideration can generate the kurtotic result in the asset market series even as the underlying process is bounded and not kurtotic. It may well be that a qualitatively similar result could arise for a system with such a set of reaction functions, driven by a purely stochastic process. But some propose that the underlying fundamentals processes are more unstable than has been thought and are subject to considerable switching behavior themselves, which can explain much of the volatility of asset markets (Evans, 1998), although the sources of such fundamentals switching remain themselves unexplained. One virtue of the model in this paper is that there is no unexplained source of switching, it being clearly defined in terms of thresholds for different agent types regarding their purchasing strategies rather than some mysterious exogenous shifts.

With regard to the second issue some recent developments offer some hope. Although many doubt that agents will be able to truly figure out chaotic dynamics, it is increasingly clear that under some circumstances agents may be able to mimic the behavior of chaotic systems by following some relatively simple adaptive rules of behavior, such as one-period lagged autoregressive processes (Grandmont, 1998). Such an outcome is called a ≅self-fulfilling mistake≅ (Grandmont, 1998) or a Aconsistent expectations equilibrium≅ (Hommes and Sorger, 1998). Indeed, even if they do not start out with the correct autoregressive process boundedly rational agents may be able to converge on one that will accurately mimic the chaotic dynamic, a phenomenon known as Alearning to believe in chaos≅ (Hommes, 1998; Sorger, 1998). Although this result was originally shown for chaotic processes generated by piecewise linear functions such as asymmetric tent maps, it is now understood that these phenomena can occur in a broader range of chaotic dynamics including those that are smooth (Hommes and Rosser, 2001), and that such learning processes may be robust and even improved in the presence of noise (Hommes and Rosser, 2001). Hence, it would appear that the problem of agents learning how to track an underlying chaotic dynamic does not seem as intractable as it has in the past.

Another point is that although a particular version of coupled flare attractors has been presented in this paper, the general qualitative results from such attractors have been shown to be robust to a wide variety of specifications. This particular specification has the advantage of being connected to some known economic models. But the generic nature of the qualitative results suggests that such models may be usable in a wider variety of specifications.

Although we have compared this model to that of Arthur, Holland, LeBaron, Palmer, and Tayler (1997), it is worth noting that it exhibits a greater degree of volatility and kurtosis than does that model, although we suspect that their model could produce such results under some different specifications regarding the strategies that derive from the expectational models of the agents involved. As already noted, the model in this paper does not explicitly involve an evolution of strategies or agent expectations, although this might be a very fruitful avenue for future research. Examples of research that might be consistent with the approach in this paper and which involve evolutionary learning dynamics include Albin with Foley (1998) and Young

(1998). In both of these models are presented of complex game theoretic group monetary dynamics in which thresholds are crossed and recrossed generating large changes in outcomes that then can revert to their earlier patterns, somewhat along the lines that the model in this paper exhibit, if not exactly the same.

Finally, we note that this model generates series with fractal characteristics (Hartmann and Rössler, 1998). Thus it provides results that resemble to some degree those developed by Mandelbrot (1997) and his associates (Mandelbrot, Fisher, and Calvet, 1997). However, in contrast with those studies there is a clearer theoretical foundation for why and where the results are coming from. Thus we see serious potential for the social flare attractor approach in analyzing these kinds of problems.

6. Conclusions

We have seen that models of socially coupled flare attractor systems can mimic the kurtotic behavior evident in many financial markets. Flare attractors have been used to model solar flare activity as well as certain autocatalytic chemical reactions. Such systems depend on underlying fundamentals processes generating chaotic dynamics. These in turn trigger nonlinear agent reaction functions that contain switching thresholds between stabilizing behavior and destabilizing behavior as well as responding to the behavior of other agents. Various agent types are coupled together having different thresholds and initial conditions. With a variety of agent types the system dynamics exhibit kurtosis but also show patterns resembling financial market series in contrast to the more starkly flaring pattern seen when there is only one agent type. Of course many alternative models and explanations remain.

Despite some questionable assumptions and aspects, this approach offers several intriguing and advantageous elements for the modeling of asset markets. One is the ability to model heterogeneous agent types that is becoming increasingly used in such analysis. This suggests the possible adaptation of such models to those with evolutionary learning. Another appealing aspect is that they generate fractal outcomes with a clear theoretical foundation. Finally, we note that their qualitative results are generic to a wide variety of specifications and forms. Thus they offer definite potential for use in a variety of dynamic economic models and systems.

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