ASPECTS OF DIALECTICS AND NONLINEAR DYNAMICS

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Abstract

Three principles of dialectical analysis are examined in terms of nonlinear dynamics models. The three principles are the transformation of quantity into quality, the interpenetration of opposites, and the negation of the negation. The first two of these especially are interpreted within the frameworks of catastrophe, chaos, and emergent dynamics complexity theoretic models, with the concept of bifurcation playing a central role. Problems with this viewpoint are also discussed.

I. Introduction

Among the deepest problems in political economy is that of the qualitative transformation of economic systems from one mode to another. A long tradition, based on Marx, argues that this can be explained by a materialist interpretation of the dialectical method of analysis as developed by Hegel. Although Marx can be argued to have been the first clear and rigorous mathematical economist (Mirowski, 1986), this aspect of his analysis generally eschewed mathematics. Indeed some (Georgescu-Roegen, 1971) argue that the dialectical method is in deep conflict with Aarithmomorphism,≅ or a precisely quantitative mathematical approach, that its very essence involves the unavoidable invocation of a penumbral fuzziness that defies and defeats using most forms of mathematics in political economy.

However, this paper will argue that nonlinear dynamics offers a way in which a mathematical analogue to certain aspects of the dialectical approach can be modelled, in particular, that of the difficult problem of qualitative transformation alluded to above. This is not the entirety of the dialectical method, which remains extremely controversial and redolent with remaining complications. We shall not attempt to either explicate or defend the entirety of the dialectical approach, much less resolve its various contradictions, although we shall note how some of its aspects relate to this more specific argument.

In particular, we shall discuss certain elements of catastrophe theory, chaos theory, and complex emergent dynamics theory models that allow for a mathematical modelling of Aquantitative change leading to qualitative change,≅ one of the widely claimed foundational concepts of the dialectical approach, and a key to its analysis of systemic political economic transformation. These approaches are all special cases of nonlinear dynamics, and their special aspects which allow for this analogue depend on their nonlinearity. We note that there are some linear models that generate discontinuities and various Aexotic dynamics,≅ e.g. models of coupled markets linked by incommensurate irrational frequencies. However, we shall not investigate these examples further. In most linear models, continuous changes in inputs do not lead to discontinuous changes in outputs, which will be our mathematical interpretation of the famous Aquantitative change leading to qualitative change≅ formulation.

Part II of this paper briefly reviews basic dialectical concepts. Part III discusses how catastrophe theory can imply dialectical results. Part IV considers chaos theory from a dialectical perspective. Part V examines some emergent complexity concepts along similar lines, culminating in a broader synthesis. Part VI will present conclusions.

II. Basic Dialectical Concepts

In a famous formulation, Engels (1940, p. 26) identifies the Alaws≅ of dialectics as being reducible to three basic concepts: 1) the transformation of quantity into quality and vice versa, 2) the interpenetration of opposites, and 3) the negation of the negation, although Engels=s approach differs from that of many others on many grounds (Hegel, 1842; Georgescu-Roegen, 1971; Ilyenkov, 1977; Habermas, 1979). Whereas Marx largely used these concepts to analyze historical change, Engels drew on Kant and Hegel to extend this approach to science. Although his discussion in *The Dialectics of Nature* was reasonably current with regard to science for the time of its writing (the 1870s and early 1880s), much of its content is seen to be scientifically inaccurate by today=s standards, and many of its examples thus hopelessly muddled and wrongheaded. Furthermore, the arguments of this book would later be used to justify the ideological control and deformation of science under Stalin and Khrushchev in the USSR, most notoriously with regard to the Lysenkoist controversy in genetics.[1]

For both Marx and Engels (1848), the first of these was the central key to the change from one mode of production to another, their historical materialist approach seeing history unfolding in qualitatively distinct stages such as ancient slavery, feudalism, and capitalism. Engels (1954, p. 67) would later identify this with Hegel=s (1842, p. 217) example of the boiling or freezing of water at specific temperatures, qualitative (discontinuous) leaps arising from quantitative (continuous) changes. In modern physics this is a phase transition and can be analyzed using spin glass or other complexity type models (Kac, 1968). In modern evolutionary theory this idea has shown up in the concept of Apunctuated equilibria≅ (Eldredge and Gould, 1972), which Mokyr (1990) and Rosser (1991, Chap. 12) link with the Schumpeterian (1934) theory of discontinuous technological change. Such phenomena can arise from catastrophe theoretic, chaos theoretic, and complex emergent dynamics models.

The interpenetration of opposites leads to some of the most controversial and difficult ideas associated with dialectical analysis. Implicit in this idea are several related concepts. One is that of contradiction, and the argument that dynamics reflect the conflict of contradicting opposites that are simultaneously united in their opposition. According to Ilyenkov (1977, p. 153), AWe thought of a dynamic process only as one of the gradual engendering of oppositions, of determinations of *one and the same thing*, i.e. of nature as a whole, that mutually negated one another.≅

Setterfield (1996) notes that contradictions may be logical in nature or between real conflicting forces, with Marx probably favoring the latter view, although it is difficult to distinguish genuine dialectical contradictions from mere differences. For Marx and Engels (1848) these real conflicting forces were the classes in conflict over control of the social surplus and of the means of production, although they also argued, as is laid out more fully in Marx (1977), that a crucial contradiction is between the forces and relations of production, united in the mode of production. This in turn fundamentally arises from the evolution of the contradiction between use-value and exchange value within the commodity itself, yet another union of conflicting opposites.

Another interpretation is that this Aunity of opposites≅ implies a negation of the idea of the Aexcluded middle≅ in logic. Thus, both AA≅ and Anot A≅ can simultaneously be true. Georgescu-Roegen (1971) makes much of this aspect in his denigration of Aarithmomorphism,≅ and interprets this as meaning that between two opposites there is Apenumbra≅ of fuzziness in their boundary in which they coexist and interpenetrate, much as water and ice coexist in slush (Ockenden and Hodgkins, 1974). Such an approach can be dealt with using fuzzy logic (Zimmermann, 1988), which in turn ultimately relies on a probabilistic approach. Georgescu-Roegen (1971, pp. 52-59) further argues that the probabilistic nature of reality itself is evidence of the fuzzily dialectical nature of reality in that truth criteria in a probabilistic world are simply arbitrary. This leads him to argue that there is a deeper contradiction between continuous human consciousness and discontinuous physical reality, discrete at the quantum level. Rosser (1991, Chap. 1) argues that this is a matter of perspective or the level of analysis of the observer.

Engels (1940, pp. 18-19) confronted the contradiction between the apparently simultaneous acceptance of discontinuity arising from the idea of qualitative leaps and of continuity arising from the Afuzziness≅ implied by the interpenetration of opposites in the dialectical approach. He dealt with this by following Darwin (1859) in accepting a gradualistic view of organic evolution in which species continuously change from one into another, while arguing that in human history, the role of human consciousness and choice allow for the discontinuous transformation of quantity into quality as modes of production discontinuously evolve.

Finally there is the idea of wholes consisting of related parts implied by this formulation. For Levins and Lewontin (1985) this is the most important aspect of dialectics and they use it to argue against the mindless reductionism they see in much of ecological and evolutionary theory, Levins (1968) in particular identifying holistic dialectics with his Acommunity matrix≅ idea. This can be seen as working down from a whole to its interrelated parts, but also working up from the parts to a higher order whole. This latter concept can be identified with more recent complex emergent dynamics ideas of self-organization (Turing, 1952; Wiener, 1961), autopoesis (Maturana and Varela, 1975), emergent order (Nicolis and Prigogine, 1977, Kauffman, 1993), anagenesis (Boulding, 1978; Jantsch, 1979), and emergent hierarchy (Rosser, Folke, Günther, Isomäki, Perrings, and Puu, 1994; Rosser, 1995). It is also consistent with the general social systems approach of the dialectically oriented post-Frankfurt School (Luhmann, 1982, 1996; Habermas, 1979, 1987; Offe, 1997).

Indeed, even some Austrian economists have emphasized self-organization arguments, with Hayek (1952, 1967) developing an emergent complexity theory based on an early version of neural networks models and eventually (Hayek, 1988, p. 9) explicitly acknowledging his link with Prigogine and with Haken (1983). Lavoie (1989) argues that markets self-organize out of chaos. Sciabarra (1995) argues that Hayek in particular uses a fundamentally dialectical approach.

Finally, the Anegation of the negation≅ has also been a very controversial and ideologically charged concept. It represents the combining of the previous two concepts into a dynamic formulation: the dialectical conflict of the contradictory opposites driving the dynamic to experience qualitative transformations. Again, there would appear within

Marx and Engels to be at least two incompletely integrated ideas. On the one hand there is the idea of a sequence of Aaffirmation, negation and the negation of the negation≅ or Athesis, antithesis, synthesis,≅ as described by Marx (1992, p. 79). This implies a historical sequence of alternating stages, with Engels (1954, p. 191) suggesting the alternation of communally owned property in primitive societies, followed by privately owned property later, with a forecasted return to communally owned property under socialism in the future.[2] On the other hand, in Marx and Engels (1848) this takes the form of one class being the thesis, the opposed class *during the same period and mode of production* being the antithesis, and the new mode of production with its new class conflict being the synthesis. We shall not attempt in this paper to resolve this contradiction, nor shall we attempt to model this explicitly in our mathematical approach.

III. Catastrophe Theory and Dialectics

The key idea for analyzing discontinuities in nonlinear dynamical systems is *bifurcation*, and was discovered by Poincaré (1880-1890) who developed the qualitative theory of differential equations to explain more-than-two-body celestial mechanics. Consider a general family of n differential equations whose behaviour is determined by a k-dimensional control parameter μ, such that

$$dx/dt = f_\mu(x);\ x\ 0\ R^n,\ \mu\ 0\ R^k, \quad (1)$$

with equilibrium solutions given by

$$f_\mu(x) = 0. \quad (2)$$

Bifurcations will occur at singularities where the first derivative of $f_\mu(x)$ is zero and the second derivative is also zero, meaning that the function is not at an extremum, but is rather at a degeneracy. At such points structural change can occur as an equilibrium can bifurcate into two stable and one unstable equilibria.

Catastrophe theory involves examining the stable singularities of a potential function of (1), assuming that there is a gradient. Thom (1975A) and Trotman and Zeeman (1976) determined the set of such stable singularities for various dimensionalities of control and state variables. Arnol=d, Gusein-Zade, and Varchenko (1985) generalized this analysis to higher orders of dimensionalities. These singularities can be viewed as points at which equilibria lose their stability with the possibility of a discontinuous change in a state variable(s) arising from a continuous change in a control variable(s).

A catastrophe form that shows most of the phenomena occurring in catastrophe models is that of the three dimensional cusp catastrophe, shown in Figure 1. In this figure J is the state variable and C and F are the control variables. Assuming that the Asplitting factor≅ C is sufficiently large, continuous variations in F can lead to discontinuous changes in J. The intermediate sheet in Figure 1 represents an unstable set of equilibria points. Behaviour observable in such a dynamical system can include bimodality, inaccessibility, sudden jumps, hysteresis, and divergence, the latter arising from variations of the splitting factor C.

For René Thom this becomes the mathematical model of morphogenesis, of qualitative transformation from one thing into something else, following the analysis of D=Arcy Thompson (1917) of the emergence of organs and structures in the development of an organism. Furthermore, Thom (1975B, p. 382) explicitly links this to dialectics, albeit of an idealist sort:

ACatastrophe theory...favors a dialectical, Heraclitean view of the universe, of a world which is the continual theatre of the battle of between >logoi,= between archetypes.≅

There is a serious criticism which can be joined of this view, although we tend to favor this view in this paper. It is the Aanti-arithmomorphic≅ dialectic position as enunciated by

Georgescu-Roegen (1971) which would argue that all we are seeing in such models is discontinuous changes in variables or functions and not a true qualitative change. The latter would presumably be something beyond the ability of mathematics to describe. It would not be simply a change in function or values of existing state variables, but the emergence of a completely new variable or even a new function or set of functions and variables. But at a minimum such structural changes imply qualitatively different dynamics, even if the variables themselves are still the same, in some sense.

Another variation on this latter point arises from considering the phenomenon of divergence associated with the change in the value of a splitting factor such as C in Figure 1. One goes from a system with one equilibrium to one with three equilibria, one of them unstable. The new equilibria themselves may actually represent new states or conditions, the qualitative change or emergence of new Avariables≅ or Afunctions≅ in some sense. This is certainly the interpretation of Thom who identified such structural changes with the emergence of new organs in the development of organisms.

Ironically, in mainstream economics most of the criticism of catastrophe theory has come from the opposite direction, claims that it is too imprecise, too poorly specified, unable to generate forecasting models with solid theoretical foundations, too ad hoc, and so forth. Much of this criticism has probably been overdone as discussions in Rosser (1991, Chap. 2) and Guastello (1995) suggest.

Another possible difficulty is that it is not at all clear that the control versus state variable idea maps meaningfully onto the dialectical taxonomy. After all, it can be argued that it is the control variables themselves that should be undergoing some kind of qualitative change as a result of their quantitative changes, rather than some state variable controlled by them.

Yet another issue that cuts across all nonlinear dynamical interpretations of dialectics is that catastrophe theory analyzes equilibrium states and their destabilization. There is an old view among dialecticians that equilibrium is not a dialectical concept, indeed that dialectics is necessarily an anti-equilibrium concept. However, drawing on the work of Bogdanov (1912-1922), Bukharin (1925) argued that an equilibrium reflects a balance of conflicting dialectical forces and that the destabilization of such an equilibrium and the emergence of a new one is the Aqualitative shift.≅ This view was sharply criticized by Lenin (1967) and was viewed by Stalin as constituting part of Bukharin=s unacceptable ideology of allowing market elements to persist as an equilibrating force in socialist society. Stokes (1995) argues that Bogdanov=s views provided the foundation for general systems theory as it developed through cybernetics (Wiener, 1961). These approaches would eventually lead to nonlinear complexity theories, some of them emphasizing disequilibrium or out-of-equilibrium phase transitions as in the Brussels School approach (Nicolis and Prigogine, 1977).

IV. Chaos Theory and Dialectics

The study of chaotic dynamics also originated with Poincaré=s qualitative celestial mechanics. As argued in Rosser (1991, Chaps. 1 and 2) catastrophe theory and chaos theory represent two distinct faces of discontinuity, and hence arguably of dialectical Aquantity leading to quality.≅ The common theme is bifurcation of equilibria of nonlinear dynamical systems at critical values.

Although there remain controversies regarding the definition of chaotic dynamics (Rosser, ibid), the most widely accepted sine qua non is that of sensitive dependence on initial conditions (SDIC), the idea that a small change in an initial value of a variable or of a parameter will lead to very large changes in the dynamical path of the system. This is also known as the Abutterfly effect,≅ from the idea that a butterfly flapping its wings could cause hurricanes in another part of the world (Lorenz, 1963).

Figure 2 exhibits this divergent behavior from small initial changes that occurs when SDIC holds. This shows the two distinct paths over time for one variable with and without a perturbation to an initial condition equal to 0.0001 for a three equation system of atmospheric circulation due to Edward Lorenz (1963). Lorenz concluded that the butterfly effect implies the futility of long-range weather forecasting. Truly chaotic systems exhibit highly erratic, apparently random, yet deterministic and bounded dynamics.

A sufficient condition for SDIC to hold is for the real parts of the Lyapunov exponents of the system to be positive. Oseledec (1968) showed that these can be estimated for a system such as (1), if $f_t(y)$ is the t-th iterate of f starting from an initial point y, D is the derivative, vΠ is a direction vector. The Lyapunov exponents are solutions to

$$L = \lim_{t \to \infty} \ln(\|Df_t(y)v\|)/t. \tag{3}$$

Although there are systems that are everywhere chaotic, many are chaotic for certain parameter values and are not for others. In such cases there may be a “transition to chaos” as a parameter value is varied and a system experiences bifurcations of its equilibria. A pattern exhibited by many well known systems is for there to be a zone of a unique and stable equilibrium, then beyond a critical parameter value there emerges a two-period oscillation, then beyond another point emerges a four-period oscillation, an eight-period oscillation, and so forth, a sequence known as a period-doubling cascade of bifurcations (Feigenbaum, 1978). According to a special case of Sharkovsky=s (1964) Theorem, the emergence of an odd-numbered orbit (>1) is a sufficient condition for the existence of chaos. In some systems, as the parameter continues to change, chaos disappears and period-halving bifurcations return the system to its original condition, although in some systems there is simply an explosion or a transition to yet other kinds of complex dynamics.

Probably the most intensively studied simple equation that generates chaotic dynamics in economic models is the difference logistic, given by

$$x_{t+1} = \alpha x_t(k - x_t) \tag{4}$$

with α being the Atuning parameter≅ whose variations change the qualitative dynamics of the system. As α increases the period-doubling cascade of bifurcations from an initial unique equilibrium described above occurs, leading to chaotic dynamics, and culminating in explosive behaviour. May (1976) studied this equation in the context of an ecological population dynamics model, in which k has the interpretation of a carrying capacity constraint, but he also first suggested the applicability of chaos theory to economic analysis in this paper. Figure 3 shows the period-doubling transition to chaos pattern for the logistic equation, with α on the horizontal axis and the system=s state variable, x, on the vertical axis.

At least two possible dialectical interpretations can be drawn from (4) and generically similar systems. One is the already mentioned idea that the cascade of bifurcations can be seen as representing qualitative changes arising from quantitative changes. A smoothly varying α, or control parameter, reaches critical points where there is a discontinuous change in the nature of the dynamics. Now, an anti-arithmomorphic dialectician can again deny that this is what is meant by qualitative change in the Hegelian sense. Yes, variables are behaving differently, but they are just the same old variables, this argument runs. But, we note that if chaotic dynamics herald a larger-scale catastrophic discontinuity, then there may be a greater chance for a deeper-level qualitative change to happen. Such instances may be Achaostrophes≅ associated with the Ablue-sky≅ disappearance of an attractor after a chaotic interlude (Abraham, 1985), or lead to Achaotic hysteresis≅ (Rosser, 1991, Chap. 17; Rosser and Rosser, 1994). Although not labeled as such, an example of such a chaotic hysteretic model is a modified Hicks-Goodwin nonlinear business cycle model due to Puu (1997) in which chaotic dynamics appear at points of discontinuous jumps in a hysteresis cycle.

The second such interpretation involves the concept of the interpenetration of opposites. This interpretation can be derived from considering the dual role of the x variable in (4). It operates both in a positive way and in a negative way, both tending to push up and to push down. Now, this may seem fairly trivial, as many such equations exist. But indeed, at the heart of most chaotic dynamics is a conflict between factors pushing in opposite directions. In effect, as α increases, the strength of this conflict can be thought of as intensifying.

In the population ecology model of May (1976), α represents the intrinsic growth rate of the population, and the negative aspect represents the effect of the population crashing into the ecological carrying capacity, k. One can view this system dialectically and holistically as a population with its environment. Conflicting forces operate through the same variable, the population, hence the interpenetration of the opposites whose interaction drives the dynamics. As this conflict heightens, bifurcations occur and quantitative changes lead to qualitative changes in dynamics as the system transits to chaos.

V. Emergent Dynamics Complexity and Dialectics

In contrast to the theories of catastrophe and chaos, there is no single criterion or model of complex dynamics, but rather a steadily increasing plethora which we shall not attempt explicate in any detail here (Arthur, Durlauf, and Lane, 1997; Rosser, 1998). Indeed Horgan (1997) reports up to 45 different definitions of complexity, including some such as algorithmic complexity in which we are not interested. Almost all involve some degrees of stochasticity in their formulation, yet some are analytical equilibrium models involving such phenomena as the spin glass models that imply phase transitions and hence could be viewed as the modern versions of the Hegel-Engels boiling/freezing water example (Brock, 1993; Rosser and Rosser, 1997). Some involve non-chaotic strange attractors, fractal basin boundaries, or other complicated nonlinear phenomena, besides catastrophe and chaos, although some of these can exhibit them as well (Lorenz, 1992; Rosser and Rosser, 1996; Brock and Hommes, 1997; Feldpausch, 1997). Virtually all of these models can be seen to exhibit the sort of dialectical dynamics associated with chaotic dynamics in terms of bifurcation points generating qualitative dynamical changes and conflicts between opposing elements driving the dynamics.

In contrast there are dissipative systems models that imply either fully out-of-equilibrium dynamics, as in the Brussels

School models (Nicolis and Prigogine, 1977) mode-locking entrainment models (Sterman and Mosekilde, 1994), the Santa Fe adaptive stock market dynamics models (Arthur, Holland, LeBaron, Palmer, and Taylor, 1997) and Aedge of chaos≅ models (Kauffman, 1993), or a temporary equilibrium that differs from a presumed long-run equilibrium as with the self-organized criticality approach (Bak, Chen, Scheinkman, and Woodford, 1993). Many of these models involve large-scale equations systems and simulations with self-organization phenomena emerging from the dynamics of conflicting forces. Such self-organization has long been identified by many observers as constituting exactly the kind of qualitative change that the dialecticians seek, and may represent overcoming the problem of the lack of new variables or functions emerging associated with the catastrophe and chaos models. All of these models can be united under the label *emergent dynamics complexity*.

However, at this point we need to step back a bit and consider how the currents involving complexity and dialectics have developed. A central point that appears is the gulf that exists between the analytic Anglo-American tradition and the Continental tradition. Urban/regional models based on the Brussels School Aorder through fluctuations≅ approach (Allen and Sanglier, 1981) exhibited polarizing outcomes and multiple equilibria long before such models became popular at Santa Fe.

In a survey of urban/regional modeling, Lung (1988) attributes this to the tradition of Adialectical discourses of French culture≅ in contrast with AAnglo-American approaches,≅ the dialectical tendency extending beyond the Germanic Hegelian base into Latin Europe as well. Indeed we have already seen this with René Thom=s willingness to put a dialectical interpretation upon catastrophe theory.

Without doubt the dialectical method/approach is in very ill repute in many Anglo-American circles, where the emphasis is upon reductionism, positivism, a narrow version of Aristotelian logic, comparative statics, and forecastibility along Newtonian-Laplacian lines. The dialectical method is viewed as unscientific, fuzzy-minded, and given to ideological mumbo-jumbo. This latter view has increased especially in economics with the increasing tendency for dialecticians in the Anglo-American economics world to be Marxists. Of course, in Continental Europe Marxist analysis tends to be more accepted, but non-Marxist dialectical approaches or interpretations are more widespread, as the discussions by Thom, Prigogine, and even the possibly dialectical element showing up in Hayek indicates. Thus, Europeans in general are more willing to admit the dialectical interpretations of emergent order and self-organization in complex dynamical systems as we have presented them above than are their American counterparts.

As a final frisson to this discussion, let us consider somewhat more closely the Stuttgart School synergetics approach of Haken (1983) that is very closely related to Prigogine=s Brussels School approach. We can see in this approach the integration of several of our kinds of nonlinear dynamics with their related dialectical interpretations. As with Allen and Sanglier (1981) and the Brussels School approach, Weidlich and Haag (1987) use the synergetics approach to model multiple equilibria and polarization in urban/regional models, followed by the analytical results of Fujita (1989) and the more recent simulation modelling at Santa Fe by Krugman (1996). Unsurprisingly, Krugman completely ignores any dialectical interpretation of the self-organization phenomenon, reflecting the Anglo-American bias.

Following Haken (1983, Chap. 12), there is a division between Aslow dynamics,≅ given by the vector F, and Afast dynamics,≅ given by the vector q, corresponding respectively to the control and state variables in catastrophe theory. F is said to Aslave≅ q through a procedure known as Aadiabatic approximation,≅ and the variables in F are the Aorder parameters≅ whose gradual (Aquantitative change≅) leads to structural change in the system.

A general model is given by

$$dq/dt = Aq + B(F)q + C(F) + 0, \qquad (5)$$

where A, B, and C are matrices and 0 is a stochastic disturbance term. Adiabatic approximation allows this to be transformed into

$$dq/dt = -(A + B(F))^{-1}C(F), \qquad (6)$$

which implies that the slow variables are determined by A + B(F). Order parameters are those with the least absolute values, and ironically are dynamically unstable in the sense of possessing positive real parts of their eigenvalues in contrast to the fast Aslaved variables.≅

This implies a rather curious possibility regarding structural change within the synergetics framework. Haken (ibid) identifies the emergence of chaotic dynamics with the destabilization of a previously stable Aslaved variable≅ as the real part of its eigenvalue passes the zero value and goes positive. Such a bifurcation can lead to a complete restructuring of the system, a chaostrophic discontinuity with more substantial qualitative implications in terms of the relations between variables, if not necessarily for their existence. The former slave can become an order parameter, and Diener and Poston (1984) call this particular phenomenon, Athe revolt of the slaved variables.≅ If this is not a dialectical outcome, then there are none in nonlinear dynamics.

VI. Conclusions

We have reviewed the three main Alaws of dialectics≅ as presented by Engels in *The Dialectics of Nature* (1940, p. 26). These are the transformation of quantity into quality and vice versa, the interpenetration of opposites, and the negation of the negation. We have seen how such nonlinear dynamical models, such as those capable of generating catastrophic discontinuities, chaotic dynamics, and a variety of other complex dynamics such as self-organization can be interpreted as manifesting these laws, especially the first two. In particular the role of bifurcation is seen as central to implying the first of these concepts, although we note that we have presented at best a very superficial overview of these various nonlinear dynamical models.

However, we must conclude with a caveat that has floated throughout this paper. Dialecticians who oppose the use of mathematical modelling at all, who identify such modelling with Aarithmomorphism≅ and a denial of essential dialectical fuzziness, will remain unconvinced by all of the above. They will see the kinds of discontinuous changes implied by the various bifurcations in these models as simply sudden changes in the values or behaviors of already existing variables, rather than the true qualitative emergence that cannot be captured mathematically. They might have a harder time maintaining such a position with regard to complexity models with self-organizing or emergent hierarchy dynamics, but even with these they can make similar arguments that one is simply seeing different behavior of already existing variables, however new and different that behavior might appear.

Of course, this hard core position is exactly that which is derided by the analytic Anglo-American tradition that sees dialecticians as hopelessly fuzzy and unscientific. The debate between these strongly held positions can itself be viewed as a dialectic that remains unresolved.

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