

COMPLEXITY IN ECONOMICS

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Introduction

The Emergence of Complexity in Economics

Increasingly in economics what had been considered to be unusual and unacceptable has come to be considered usual and acceptable, if not necessarily desirable. Whereas it had been widely believed that economic reality could be reasonably described by sets of pairs of linear supply and demand curves intersecting in single equilibrium points to which markets easily and automatically moved, now it is understood that many markets and situations do not behave so well. Economic reality is rife with nonlinearity, discontinuity, and a variety of phenomena that are not so easily predicted or understood. At the same time the broad coherence of economic systems is more impressive than ever in the face of such phenomena. The order of the economy appears to emerge from the complex interactions that constitute the evolutionary process of the economy.

These phenomena have come to be labeled as *complexity in economics*. Even what seems simple in economics generally arises from behavior not reflecting rational expectations; we live in a world that reflects the enormous variety and diversity of humanity in their knowledge, attitudes, and behaviors, interacting with each other in an enormous range of institutional frameworks. What emerges in the aggregate may have little to do with what happens at the individual level. But this aggregate cannot be simply described by some set of aggregate equations. It emerges out of the soup of the individual and particular with all its multiform interactions and peculiarities.

This change of perspective has brought forth a variety of new approaches to analysis. Previously a premium was placed on deductive formal proofs of theorems that sought to derive general solutions broadly applicable, a view that reached its culmination in the French Bourbakist school. Now we see a greater emphasis on computer simulations and experimental methods to inductively determine possible outcomes and ranges of solutions. Emergent phenomena from complex systems are not usually discovered by theorems but more frequently by the use of increasingly powerful computers to explore the limits and possibilities that can arise. The new awareness of the ubiquity of complexity is transforming the way that we think about economics.

What is Economic Complexity?

The idea that reality reflects complexity did not initially arise in economics. It has come from numerous disciplines of thought and has reverberated back and forth across them, from mathematics and computer science to physics and biology to the social sciences and back and forth. That different disciplines have come to see it as meaning something somewhat different from what other ones do is not surprising. Indeed, the physicist, Seth Miller, has gathered at least 45 different definitions of “complexity.” Many of these are not appropriate for economics, or only barely so for special occasions or circumstances. Computer science is the origin of many of these, and arguably many of these are among the more rigorous of such definitions, such as the length of an algorithm for solving a specific problem. Such ideas had important influence on economic thinking through such figures as Herbert Simon, an important figure in the development of artificial intelligence in computer science as well as a Nobel Prize winner in economics

for his conceptualization of bounded rationality. But this is not what most economists think of when they think of complex economic dynamics, although reality of complexity in economics has come full circle to Simon in implying the inevitability of bounded rationality. Economic complexity is not simply a matter of things being “complicated,” although some have viewed it this way, and certainly economic reality is both “complicated” as well as *complex* in the sense that has increasingly come to be accepted as most relevant for economics.

At the risk of oversimplifying complexity, we shall follow Richard Day (1994) in defining complexity in economics in terms of dynamic outcomes. An economic system is dynamically complex if its deterministic endogenous processes do not lead it asymptotically to a fixed point, a limit cycle, or an explosion. Some might argue that endogenous limit cycles also constitute complexity, but we shall view them as merely nearly complex. We note that all systems that fit this definition have some degree of nonlinearity within them, however limited or arbitrary. At the same time there are nonlinear systems that are not complex, such as a standard exponential growth model.

Now we observe that this is a “broad tent” definition of complexity that includes systems that some would argue should not be included. There is a “narrow tent” view of economic complexity, but it lacks a simple or clear definition. It can only be characterized by fitting with a set of characteristics. Arthur, Durlauf, and Lane (1997) provide such a list of characteristics. They include 1) dispersed interaction among heterogeneous agents acting locally on each other in some space; 2) no global controller that can exploit all opportunities or interactions in the economy even though there might be some weak global interactions; 3) cross-cutting hierarchical organization with many

tangled interactions; 4) continual adaptation by learning and evolving agents; 5) perpetual novelty as new markets, technologies, behaviors, and institutions create new niches in the ecology of the system, and 6) out-of-equilibrium dynamics with either zero or many equilibria existing and the system unlikely to be near a global optimum.

How do these two apparently competing perspectives relate to each other and how are we going to deal with them in this work? The second will generally be a subset of the first (although there are certainly systems that could fulfill all six of these characteristics and yet violate the first definition). More accurately it can be argued that the second is the historical outcome of the first. In his essay reproduced in Section I, John Horgan (1995) criticizes the concept of “complexity” more generally, not just in economics, as just the latest in a string of fads, “the four C’s.” In his view these four C’s are *cybernetics, catastrophe, chaos, and complexity*. This reflects the approximate order of their appearance as broad, transdisciplinary fads, with cybernetics hot in the 1960s, catastrophe theory hot in the 1970s, chaos theory hot in the 1980s, and complexity coming in during the 1990s. In Horgan’s view each of these was overhyped, was a sort of intellectual bubble that blew up and then crashed. For him complexity is just the latest of these and will crash also.

The view of this introduction is to take Horgan’s argument seriously but to turn it on its head. First, he is correct to identify intellectual linkages between these intellectual movements. They all are a part of *complexity* in our “broad tent” sense. Not only that, but they have to some degree arisen out of each other, even though each has specific sharp differences from the others. This is even reflected in that certain individuals have

played roles in various of these stages, with a few even arguably having been involved in all four. There is an intellectual unity to “broad tent” complexity.

Second, he is wrong to have viewed them as having simply disappeared when they “crashed” after their admittedly respective overhypings (although it is not clear that the last, “narrow tent” complexity, has actually “crashed,” despite various doubts and caveats having been raised about it). Cybernetics lives on in the newer complexity through the use of simulation and the idea that “counterintuitive” results can arise from simulating systems of nonlinear differential equations. Catastrophe theory, perhaps the most criticized and “academically unacceptable” of the four, persists in the widespread perception of the reality of profound discontinuity, whether it is the crash of a stock market or a currency, or the collapse of an ecologic-economic system. The newer complexity theory of dispersed agents has discovered its own ways of generating this outcome of the older theory. The situation with chaos theory is more complicated as its progenitors and practitioners are even more closely linked to the newer views. This fact is recognized by Horgan in his label of *chaoplexology*, which may be a non-category, but whose very expression gives credence to the idea of the unity of the stream of ideas that have been building over several decades to reach a critical intellectual mass that is now more profoundly altering the views of economists more widely.

Our approach will be to present papers that tend to emphasize the more recent viewpoint, which is clearly what is generally driving more recent research. However, we shall also include certain papers that represent the earlier perspectives of the broader definition of economic complexity that we have presented. This will allow for both historical accuracy and also to show the essential unity of these approaches as many

papers will mix various of these supposedly distinct viewpoints. However the book will be organized by areas of economics. Thus within each area there will be papers of various of the perspectives, although generally with a greater emphasis on the more recent view of what economic complexity is. We note however that we shall not include any papers that have already appeared in the volume on chaos theory edited by Dechert (1996), although some of those would have been candidates for this volume.

We shall now proceed with a summary of the papers presented in their ten categories.

Philosophical and Methodological Overviews

The first paper dates from 1962, long before the current discussions of the meaning or nature of complexity in economics. Indeed it is not explicitly about economics at all. And even though its author, Herbert Simon, would eventually receive a Nobel Prize in economics, he was trained as a political scientist and was in the departments of computer science and psychology at Carnegie-Mellon, not economics. He received his Nobel Prize for developing the concept of *bounded rationality*, and for that alone he deserves to have first place in this compendium of papers about Complexity in Economics. This paper does not fit well with the definitions presented above, but it has been very influential across many disciplines. Certainly it implies Simon's bounded rationality idea, that appears also to be implied by all the various schools of modern economic complexity.

The second, by Peter Albin, deals with logical problems arising from the infinite regress implied by trying to think about how other people are thinking about how we are thinking about how they are thinking about how we are thinking about how they are thinking about... Of course the Nash equilibrium was supposed to resolve all that, but it

only does so to a limited extent. Again, arguably Albin's concerns look more like those of theoretical computer science or logic, but they imply the problems that have come to be taken more seriously in a world where we admit that agents are heterogeneous and we must take seriously the explicit modeling of their interactions, and the interactions of their thinking about their interactions, a problem that is ultimately unresolvable as Albin argues, following such figures as Gödel and Turing before him.

The third paper by Don Lavoie presents the Austrian view that the idea of emergent order out of complexity explains the phenomenon of the spontaneous order of free market systems. Lavoie's arguments draw heavily on earlier arguments by Friedrich Hayek (1948, 1967) who was one of the first thinkers in any discipline to consider seriously the problem of complexity.

The fourth piece in this section is Horgan's original statement of the argument discussed above. For a more extended discussion of his argument, see Horgan (1997). The following piece by Rosser responds in more detail to him, as well as containing a broader discussion of the issues as they relate to economics.

Social Interactions and Learning Dynamics

This section deals directly with core issues for the newer view of economic complexity, specifically the issue of locally interacting heterogeneous agents.

Although arguably the first paper to examine local interactions by heterogeneous agents was that by Schelling in 1971, presented in Section VII, it was in some sense an ad hoc exercise. The first formal analysis of interacting heterogeneous agents in economics is the first paper in this section, by Föllmer in 1974. It drew explicitly on the theory of

interacting particle systems from statistical mechanics in physics, which would inspire much later work in this area. It is a dramatic example of a paper far ahead of its time, only generating influence a quarter of a century later, although some of those it influenced would point out its limitations.

The year 1993 was when the direct descendants of that paper appeared, with the second one in this second section being an important example. Blume especially focuses on game theoretic issues involved.

Kirman's paper from the same year has influenced study of financial markets as well as being a general model of heterogeneous interactions. Kirman has been among those most forcefully criticizing the idea of assuming an economy reflects the behavior of a representative agent. This paper has been his most influential proposed alternative and suggests the theme of biology and evolution in economic complexity.

The paper by Brock and Hommes is one of those that shows the overall links between the different perspectives on economic complexity. While explicitly modeling heterogeneous agents, if analytically rather than computationally, it shows how the dynamic interactions of such agents responding to each to each other can generate a variety of unusual phenomena including mathematical chaos almost as a minimum. If there is an economist who deserves the label "chaoplexologist," it may well be William A. (Buz) Brock.

The paper by Kollman, Miller, and Page is an example of computational political economy, and is included here because its use of simulated annealing techniques, one of the interacting particle systems models borrowed from physics, allows for the search for a better solution in a world of multiple equilibria when a system locates itself on a

suboptimal local equilibrium. The “heating” in the annealing process allows for a searching for globally better solutions.

Grandmont’s paper from 1998 is the published version of an influential lecture he gave to the Econometric Society in 1990. Thus its placement does not reflect its real influence or prominence in the development of ideas. It is the original paper that suggests that learning processes can pursue a false model that nevertheless reproduces reality in some sense. This would inspire the idea of *learning to believe in chaos* discussed in the following paper by Hommes and Sorger, the idea that persons following simple rule of thumb behaviors might be able to learn to accurately track complex underlying realities.

Hartmann and Rössler are chemists who adapt their idea of flare attractors, used to explain solar flares and various catalytic chemical reactions, to study the interaction of heterogeneous economic agents in discrete dynamical settings. Their specific context is entrepreneurship and the formation of new businesses, but this idea has clear implications for a variety of economic settings, especially in financial markets (Rosser, Ahmed, and Hartmann, 2003).

Brock and Durlauf’s paper is another like Grandmont’s that originally appeared in unpublished form much earlier in 1995 and has had an enormous influence since. It is arguably the definitive presentation of the mathematics of social interactions between heterogeneous agents in economics within a discrete choice context.

Competitive Market Dynamics

The lead author of the first paper in this section, Mark Granovetter, is a mathematical sociologist famous for analyzing the implications of interactions between agents. Here with Ronald Soong he considers old, but generally forgotten, economics models of consumers reacting to each other both positively in bandwagons and negatively as snobs and derives both chaotic and more generally complex dynamics in a fascinating deconstruction of the demand curve of standard economics when we seriously consider the interactions between demanders in the market.

The next paper, by Carl Chiarella, shows how lags can lead to chaotic dynamics even with monotonic supply and demand curves in a simple competitive market model. This cobweb model has been much studied by many economists and some have argued that the first discussion of chaotic dynamics appears in the paper by Mordecai Ezekiel (1938) on the cobweb. It is true that Ezekiel recognized that cobweb dynamics could become more irregular than the simple convergences, explosions, or limit cycles that he explicitly depicted, and therefore it can be argued that he was an early prophet of complexity. But his discussion does not clearly imply chaotic dynamics as such.

Duncan Foley presents one of the first models using statistical mechanics in a competitive market framework, introducing an entropy minimization condition as the organizing principle of price determination in a stochastic market context. His results are consistent with approaches emphasizing power laws and econophysics. He has since identified these power law outcomes as potentially explaining inequalities in wealth and income in market economies.

Donald Saari's piece for the *Notices of the American Mathematical Society* shows how for even the simplest of competitive models a wide array of possible complexities can arise. In this paper he argues that, "[W]hat we know indicates that even the simple models from introductory economics can exhibit dynamical behavior far more complex than anything found in classical physics or biology" (p. 222). It is the added complexity arising from the interactions among human decision makers that leads to this result.

Goeree, Hommes, and Weddepohl show the complexities that can arise using the most standard of Walrasian disequilibrium adjustment mechanisms, the *tâtonnement* process.

Dynamics of Imperfect Competition

The paper by David Rand was the first to consciously present chaotic dynamics in an economic model, although the ecologist/physicist, Robert May, had earlier (1976) suggested such a possibility. Rand's model studying duopoly dynamics within the Cournot framework is ironic in that when Cournot himself first presented his model (1838) it was the first application of calculus to an economics problem. Rand's results depended on non-monotonic reaction functions with multiple equilibria.

Bonanno follows a long literature derived from the original observation of Joan Robinson (1933, pp. 57-58) that marginal revenue curves can easily be non-monotonic in segmented markets. The possibility of dynamic discontinuities in price arising from continuous changes in demand in a situation of multiple equilibria implied by this case is explicitly placed in the context of catastrophe theory by him.

Puu follows up Rand's model by showing that his results can arise from simple lags in response even in the case where there is a single Cournot-Nash equilibrium. Kopel follows this up to consider global dynamics of this model and to examine various other kinds of complex dynamics.

In a Santa Fe Institute Working Paper, Robert Axtell follows the inspiration of Herbert Simon who anticipates a major idea of *econophysics* in discussing the significance of the empirical distribution of firm sizes as following a scaling character. Axtell provides an argument based on the instability of local Nash equilibria in the context of new firm formation with local economies of scale. He is able to derive results using simulation methods that reproduce important empirical phenomena.

The paper of Agliari, Bischi, and Gardini summarizes a major vein of research on global dynamics of imperfectly competitive models with a variety of dynamically complex outcomes such as multistability of attractors and riddling of basins of attraction, among others studied.

Macroeconomic Fluctuations and Growth

This section opens with the oldest paper in this collection, dating from 1953 by Strotz, McAnulty, and Naines. It is the first economics paper to clearly present actual mathematically chaotic dynamics, although the authors did not realize at the time what they had discovered other than that it was something "irregular." Given the conventional wisdom of the economics profession of the time such results were best thought to be downplayed if not to be simply suppressed. It is unsurprising that this paper has long been ignored, despite its historic primacy.

The Forrester paper is probably the only one in this collection that can be identified firmly with the first of our “four C’s,” *cybernetics*. Jay Forrester, inventor of the flight simulator in 1944, probably the first truly interactive human-computer system, has been the most important advocate of this view in economics and has inspired a group of researchers at the MIT Sloan School of Management to follow him, some of whom such as John Sterman have arguably followed through all the way on all the four C’s. In this paper he shows how interactions between multiple time scales in a nonlinear model can lead to long wave cycle outcomes on top of shorter wave cycles. This is arguably a borderline complexity outcome according to our definition. But the idea that higher order longer cycles can arise out of shorter period cycles would seem to qualify this paper.

Varian’s paper was probably the most widely cited example of using catastrophe theory in economics, making it almost semi-respectable before the backlash against this particular approach sank in seriously. Whatever one thinks of catastrophe theory, this paper remains an excellent examination of endogenous macroeconomic fluctuations arising within a multiple equilibria framework due to empirically reasonable nonlinearities in driving functions.

Lorenz’s seminal paper is the first to show the possibility of non-chaotic strange attractors and also of fractal basin boundaries in any economic model. He discovered these outcomes while studying a business cycle model that can also generate chaotic dynamics as well, based on work by Kaldor (1940) that also inspired the paper by Varian. Lorenz’s paper also brings out the importance of the idea of *transiency*, that systems can behave in a chaotic manner for periods of time and then shift endogenously into completely non-complex forms of behavior for other periods of time.

The physicist, Per Bak, has been the leading advocate of the idea of *self-organized criticality* as a generally important concept in many disciplines (Bak, 1996). This paper with his frequent coauthor Chen and the economists José Scheinkman and Michael Woodford is the most important application of this idea in economics. It shows that even if exogenous shocks are normally distributed, endogenous outcomes may be skewed, following a power law that has fractal scaling properties. Such ideas have been very influential in econophysics, although specific applications in economics of this approach have been relatively rare. The model's results depend on the concatenation of shocks through multiple stages of production in a lattice relationship.

Durlauf uses the theory of interacting agents to generate multiple equilibria outcomes in long run growth path outcomes. Differing degrees of external economies among agents are crucial to his results.

Aoki's paper is an innovative application of the mean-field approach of interacting particle systems from statistical mechanics in macroeconomics. This paper has definite links with the paper of Durlauf as well as work of Brock.

Although not the first application of the idea of controlling chaos in economics, the paper of Leo Kaas is the first to utilize both of the most widely studied methods in an intriguing manner, one more global to bring the system within range of the desired outcome and the shorter range approach to then precisely stabilize the system, although Kaas is aware of the extreme knowledge requirements for implementing such approaches in actual policy settings.

Financial Markets

The paper by Christopher Zeeman was the first published application of catastrophe theory in economics. When Zahler and Sussman (1977) engaged in their broad assault on catastrophe theory they singled out this paper for criticism on the grounds that its assumption of heterogeneous agents, fundamentalists and chartists, violated the then dominant assumption of rational expectations. By today's standards, Zeeman's approach seems prophetically up-to-date compared to the kinds of models Zahler and Sussman preferred, despite some problems that it has. Although most observers have avoided using its terminology, it remains the case that this paper and catastrophe theory have much more to say about the nature of major crashes of stock and other asset markets than do many currently popular alternative approaches.

Day and Huang's paper is the first to show the possibility of endogenously generated *chaotic bubbles* in asset markets, a term due to Rosser (1991, p. 291). They follow Zeeman in having heterogeneous agents, with a third category of market makers along with the fundamentalists and the trend chasing chartists who destabilize the markets.

Stutzer introduces entropy concepts related to econophysics notions that have become influential recently. This approach readily generates power law kinds of outcomes, but involves some different elements than the related paper by Foley in this collection.

The Arthur, Holland, LeBaron, Palmer, and Tayler paper presents a model developed at the Santa Fe Institute and long studied there. Here we have the shift from the Zeeman and Day-Huang approaches to one of a variety of heterogeneous agents interacting and learning new strategies according to an evolutionary process. We see periods when the market is fairly stable, oscillating within a narrow range around the fundamental rational

expectations equilibrium, and periods when it is more divergent and more volatile in its dynamics. Broadly these periods reflect alternating dominance by agents who are more or less “fundamentalist” in approach versus those dominated by agents who are more prone to trend chasing behavior of one sort or another, that is, de facto “chartists.”

Ultimately this paper confirms the insights of Zeeman and Day-Huang.

The paper by Thomas Lux is a major example related to the econophysics approach, based on interacting heterogeneous agents and showing how such well known phenomena in financial markets as leptokurtotic “fat tails” can arise in returns distributions, as well as a variety of complex dynamics.

The Calvet and Fisher paper is a prime example of a competing effort to explain asset market volatility using multifractals. This approach is arguably also a species of econophysics as well as of economic complexity. It is ultimately drawing on ideas due to Benoît Mandelbrot (1997). This approach implies deep structural relationships arising from the interactions of the agents.

International and Transitional Economic Dynamics

Although not invoking complexity or any of its variations, Krugman’s paper on multiple equilibria in foreign exchange markets and the possibility of sudden shifts in the value of currencies certainly invokes and even precedes his later work in the area of complex dynamics in the urban and regional areas.

Arifovic provides a more straightforward application of one of the newer complexity approaches, that of genetic algorithms that can learn and adapt over time, to the problem of modeling exchange rates, which are notoriously difficult to model well.

Ramsey and Zhang examine the dynamics of foreign exchange rates using the innovative econometric technique borrowed from physics of waveform dictionaries. This approach allows for greater flexibility and subtlety of time series relationships among variables.

Rosser and Rosser consider problems of systemic transition using the mean field approach of statistical mechanics to model the breakdown of institutional frameworks and the broader implications for macroeconomic collapse that can happen.

Baldwin, Martin, and Ottaviani present a bold interpretation of world economic history in which symmetry breaking occurs as global transportation costs decline. This implies that a semi-catastrophic discontinuity around the time of the industrial revolution in Britain triggered a global bifurcation with a resulting decline of output in India and China relative to Europe in a globalized economy. One implication of the paper is that eventually there might be a reconvergence of outcomes in an increasingly globalized economy through international trade as ease of diffusing of technology increases.

Rosser, Rosser, Guastello, and Bond examine the role of *chaotic hysteresis* in the fluctuations of investment under the old system in Russia and in its eventual systemic collapse. The idea of chaotic hysteresis involves a combination of catastrophe and chaos theory concepts and was originally due to Abraham and Shaw (1987). Such an approach was first applied in economics by Puu (1990) who used it in a multiplier-accelerator model of macroeconomic fluctuations.

In the final paper in this section, with its lead author one of the leading architects of the euro, Paul de Grauwe and his coauthor Grimaldi show how chaotic dynamics can arise in foreign exchange markets out of the interaction of fundamentalist and trend-

chasing chartist traders. This result is consistent with several of those observed earlier in the financial markets section where such interactions between heterogeneous agents could bring about a variety of possible dynamically complex outcomes.

Urban and Regional Systems

Given its inherently spatial nature it is not surprising that it has been in the area of urban and regional economics that complexity approaches have had many of their first applications, with people studying the local interactions of dispersed heterogeneous agents, even if many of these studies have not been widely recognized.

One paper now recognized as profoundly innovative, and arguably the first example of the newer complexity approach, is that of Thomas Schelling in 1971 in the *Journal of Mathematical Sociology*, its outlet showing the general attitude to the idea of social interactions among economists at the time. In contrast to current work, Schelling's simulation study was done by hand calculation and not by computer. He discovered a form of emergent structure, racial segregation in cities, given a small change in initial conditions, a slight preference by members of groups to be with those of their kind. The emergence of a major overall effect from a slight initial change is a general characteristic of complexity models in general. In cybernetics it was the counterintuitive results; in catastrophe theory it was the discontinuity at a critical value of a control parameter, and in chaos theory it was the sensitive dependence on initial conditions, more popularly known as the *butterfly effect*, the idea of Edward Lorenz that a butterfly flapping its wings in Brazil could trigger a hurricane in Texas (Lorenz, 1993).

Tönu Puu's paper clearly lays out how structural reconfigurations of regional economic systems can be studied using catastrophe theory. Just as the newer complexity view first appeared in this area of economics, so catastrophe theory was more widely used here as well, with arguably the first paper using it appearing at the same time as Zeeman's one on stock markets (Amson, 1974).

Many of these papers appeared in non-economics journals and have received little recognition among economists, despite more recent attention to the ideas that they contain. That of Allen, Engelen, and Sanglier also shows how order can emerge from a set of dispersed interactions, this time in an urban economics setting. The authors were associated with the "Brussels School" led by the Nobel Prize-winning physical chemist, Ilya Prigogine, one of the most important progenitors of broadly based complexity ideas across many disciplines (Nicolis and Prigogine, 1989).

Physicists Wolfgang Weidlich and Gunter Haag examine the phenomenon of discontinuous regional structural reconfiguration as crucial parameters vary. This paper predates later similar results by others that have been widely touted. Weidlich and Haag have been associated with the *synergetics* approach of Hermann Haken (1983) at the Stuttgart Institute of Theoretical Physics, another fountainhead of transdisciplinary complexity modeling.

Paul Krugman has been the most prominent figure associated with what has been called the *new economic geography* since 1991. This paper provides a summary of his views on this approach to complex dynamics in spatial economic systems. The following paper by him with Fujita and Mori shows how hierarchical urban systems can evolve.

Urban hierarchies have long fascinated those interested in scaling and power laws that seem to fit well the empirical data.

Evolutionary Economic Dynamics

Although there has clearly been an enormous influence in terms of techniques from physics on the recent complexity approaches, many argue that the more important ideas and approaches do, or should, come from biology. One influence from biology has been an emphasis on evolutionary dynamics, although there has long been a strand of economics that has emphasized such a view. One of the outcomes of the newer views has been to allow for more explicit mathematical underpinnings for such ideas as *path dependence* of technologies and institutional forms as well as the ability to model higher level dynamics over multiple level time scales such as Forrester considered. The rise of applications in evolutionary game theory has also been a prominent recent development.

Richard Goodwin was long an innovator of models that either themselves clearly generated complex dynamics or were shown to do so by others, as with the example of Strotz, McAnulty, and Naines discussed above. In his paper here he shows multiple scale long wave dynamics that include chaotic dynamics in an evolutionary model.

The most famous paper of Brian Arthur, highlighted in the book on complexity by Waldrop (1992), was another that had a long life in unpublished form while being highly influential. Here he shows how path dependence for technologies can arise in industrial settings. This result has had wide influence in many settings and applications.

The Day and Walter paper can be seen as an extension of the Goodwin approach with the sources of the fluctuations at different time scales more clearly drawn out. Again,

simple fluctuation patterns at one time scale interact with more complex ones at other time scales.

There follow three papers focusing upon evolutionary game theory with that by Foster and Young having received insufficient attention or credit among economists for its seminal presentation of the idea of stochastic stability of long run evolutionary outcomes. Undoubtedly this has happened because of its initial appearance in a biology journal, although economists have become more inclined to both publish in and read such journals in recent years.

Physicist Kristian Lindgren provides a fascinating account of the long run dynamics of prisoner's dilemma game theoretic outcomes with heterogeneous agents learning different strategies within different environments. One is the mean field environment drawn from statistical mechanics while the other is the self-organized criticality environment of Per Bak. In both cases one observes no settling down to any particular equilibrium configuration, with indeed occasional major restructuring of the situation and the complete disappearance of previously dominant strategies.

Ken Binmore and Larry Samuelson take things to the next level implied by this result of Lindgren by examining the long run distribution of equilibrium outcomes as evolutionary drift allows systems to move from one basin of attraction to another and one can see which basins and equilibria are more likely to occur.

Ecologic-Economic Systems

As with urban and regional systems, those studying ecologic-economic systems have long been much more open to the ideas of complex dynamics than have most economists,

with the collapses of biological populations much on the minds of many. Indeed as noted, important ideas of complexity came into economics from biology, especially from Robert May (1976), but from others as well.

The Dixon and Jones paper shows an obvious application of catastrophe theory to the collapse of the blue whale population in the 1960s. Conklin and Kolberg observe the possibility of chaotic dynamics for the halibut population, arising from the backward-bending supply curve known to be possible in fisheries under open access or high discount rate conditions for some time (Copes, 1970).

Rosser, Folke, Isomäki, Perrings, and Puu study various ways in which ecological and economic systems interact within multiple level hierarchical contexts. The idea of “cross-cutting hierarchies with tangled interactions” is a main theme of this paper, with several different models of resulting combined hierarchical systems being proposed and different mechanisms for the emergence of novelty or discontinuity implied by each.

Brown and Roughgarden show a variety of complex dynamics arising from simulations of reasonably specified interacting ecologic-economic models.

The Chen paper shows how interactions between the global economy and global climate could generate a chaotic dynamic in the global climate that interacts with a chaotic dynamic in the global economy.

Ecologists Carpenter and Ludwig and economist Brock examine management problems for the case of an ecosystem impacted by human generated pollution. Their canonical example is the much-studied Lake Mendota in Madison, Wisconsin, which has received large amounts of phosphorous fertilizer runoff from nearby corn (maize) farms.

The model of multiple equilibria with the possibility of discontinuous shifts looks much like a catastrophe theoretic approach, although they do not explicitly mention this.

The paper by Rosser analyzes and summarizes a wide variety of complex ecologic-economic dynamics models and their associated economic policy issues.

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