

Benoit B. MANDELBROT

Home

Vita, Awards, and Publications

- One-page Vita [DOC]
- Several-page Vita including awards and citations [DOC]
- Lists of books [DOC]
- Research publications other than books [DOC]
- Expository publications and the like [DOC]
- Full vita, including items not listed separately [DOC]

Books by BBM

- Printed books/translations
- Virtual books

Chronicles

- Books on fractals [DOC]
- Meetings on fractals [DOC]

Scrapbooks of Reviews

- Reviews of "fractals" books [DOC]
- Reviews of "selecta" books [DOC]
- Reviews of The (Mis)behavior of Markets [DOC]
- Reviews of other books [DOC]

Interviews

Photographs

Japan Prize 2003

Links

- Links to Festschrifts and the Like [DOC]
- Links to Books and Videos
- Links to Sites

Benoit Mandelbrot

FRACTAL AND MULTIFRACTAL FINANCE Crashes and Long-Dependence

VIRTUAL SELECTA

This open-ended webbook will be extended as needs and opportunities arise. It consists in links to the author's publications on economics and finance that are found in this home site <http://.math.yale.edu/mandelbrot>

FOREWORD

The variation of financial prices has been the topic of a large number of papers I published since 1960 and further papers are on the way. My successive models are now called *mesofractal*, *unifractal* and *multifractal*.

One very good thing is that they have been widely adopted, though all often under different names. In 1964 my stress on the importance of crashes, even of instantaneous discontinuities, was called "revolutionary" but now it has become almost banal.

A more general good thing resides in the wide recognition, today, that my goal did not limit itself to technical details or to complicated mathematics unfamiliar to "practical folks." To the contrary, my main contribution can almost be called qualitative. Indeed, between 1960 and 1965, I introduced into economics a special form of the notion of invariance that later became essential in physics and took the term "scaling." In 1972, I introduced multifractals by replacing scaling by multiscaling, a notion whose relevance to finance I recognized and mentioned immediately. Throughout, I showed that scaling and multiscaling have many practical consequences.

Those invariances and their old and new consequences are described in a book I am writing with Richard L. Hudson. Deliberately, that book is effectively free of the tedious scholarly apparatus of formulas and references. Their absence created, in turn, the urgent need of a single source for links to all the necessary backup material. The first role assigned to this "webbook" is to provide those links. This role can be said to concern the present.

A different role that can be said to concern the future is suggested by the subtitle that describes this webbook as "open-ended." This means that I intend and hope to keep it up-to-date by including links to additional material as it becomes available and posted on the web. Knowing that its publication is backed up by this web-book, it will be made far less difficult to prepare this additional material.

Yet another role concerns the "past," more precisely, the relation between this webbook and my printed books *Selecta E* (1997), *Selecta N* (1999), and *Selecta H* (2002). These volumes' idiosyncratic format combined old and new material and was motivated by my intention to move to other activities. But I didn't, hence - to my great disappointment - all that extraordinary effort turned out to have been ill-inspired. The new material was less widely accessible in a book than it would have been in periodicals. Moreover, its substance was soon developed in numerous publications that are either included in this webbook or candidates for future inclusion.

These experiences and the advent of the web will make *Selecta C* (2004) the last of the series. But this webbook provides an opportunity to correct several past mistakes.

Firstly, *Selecta H* devoted its Chapter 30 to finance. But no economist will seek out that particular book for the sake of a single chapter. To correct this miscalculation, this chapter is reproduced in this website.

Secondly, Chapter 37 of my 1982 *Fractal Geometry* is titled "Scaling and Price Change in Economics." This chapter acquainted innumerable readers with my work in finance. More recent readers, of course, have many other sources at their disposal, not all of them reliable. In any event, that chapter was influential therefore a copy has been inserted.

Thirdly, several papers from *Selecta E*, originally published in the 1960s, have been or will be scanned and included in this webbook. For several short ones written in (excellent) French, photographic reproductions were already available, but only in *Selecta* book EF in French. They have been scanned and included as such, both in this webbook and in webbook F.

Last but not least, the *Selecta* books reprint old papers written in English. The fact that original journals chose not to copy-edit them suggests that their style raised few objections. Since then, however, my standards have become more demanding. While they were reset electronically they were also extensively copy-edited by experts in English. In parallel, the original texts have become quite popular and have been reproduced, not only in my *Selecta*, but also in other collections of classic papers.

The originals are included in pdf format in this book. Therefore the reader is now presented with the choice between the original clumsy English reproduced in this webbook *Selecta*, and the copy-edited English in the printed *Selecta*. I want all that copy-editing to be worthwhile, hence recommend the latter.

Last remark: this is not the only webbook on this website. Another is devoted to multifractals. A third is open-ended and is almost completely an organizational or taxonomic fiction, since it is defined as linking all my publications written in French, most of them containing little or no mathematics. The all-encompassing title is *Essais et Portraits. Essais sur les fractales et portraits de quelques hommes de science*.

TABLE OF CONTENTS

P. Is there persistence in stock price movements? *Seminar on the Analysis of Security Prices*, held November 12-13, 1966 at the Graduate School of Business of the University of Chicago.

[[PDF](#) (52.6 KB)],

43

P. E19. M 1966b. Forecasts of future prices, unbiased markets and "martingale" models. *The Journal of Business of the University of Chicago*: **39**, 242-255.

[[PDF](#) (160 KB)]

- Reprint: *Forecasting Financial Markets*. Edited by Terence C. Mills. *The International Library of Critical Writings in Economics*. Series Editor: Mark Blaug. Cheltenham, UK: Edward Elgar, 2002.

48

P. E15. M 1967j. The variation of some other speculative prices. *The Journal of Business of the University of Chicago*: **40**, 393-413.

[[PDF](#) (968 KB)]

- Reprint: *Classic Futures: Lessons from the Past for the Electronic Age*. Edited by Lester Telser. London: Risk Books. 2000, 685-708.

54

M 1969e. Long-run linearity, locally Gaussian processes, H-spectra and infinite variances. *International Economic Review*: **10**, 82-111.

[[PDF](#) (61 KB)]

- Abstract: Intermittency and periodicity, and the problem of long cycles. *Econometrica*: **34**, 1966 (Supplement) 152-153.

59

P. M 1970e. Statistical dependence in prices and interest rates. *Papers of the Second World Congress of the Econometric Society*, Cambridge, England.

[[PDF](#) (61 KB)]

- Summary: Analysis of long-run dependence in time series: the R/S technique. *Fiftieth Annual Report of the National Bureau of Economic Research*, 1970, 107-108.
- Abstract progress report: Long-run interdependence in price records and other economic time series. *Econometrica*: **38**, 1970, 122-123.
- Reprint of Part I: Statistical dependence in prices and interest rates. *Fifty-first Annual Report of the National Bureau of Economic Research*, 1971, 141-142.
- Revised Part II: Analysis of non-periodic long-run dependence using the robust statistic R/S. *Proceedings of the 1971 Princeton Conference on Information Sciences and Systems*, 155-159.
- Analysis of long-run dependence in economics: the R/S technique. *Econometrica*: **39**, 1971 (July Supplement), 68-69.

65

M 1972c. Statistical methodology for non-periodic cycles: from the covariance to R/S analysis. *Annals of Economic and Social Measurement*: **1**, 257-288.

[[PDF](#) (61 KB)]

- Advance summary: Comment on "Application of linear random models to four annual streamflow series" by Carlson, MacCormick, & Watts. *Water Resources Research*: **7**, 1971, 1360-1362.

164

M. & P. M, Laurent CALVET, & Adlai FISHER 1997. The multifractal model of asset returns. *Cowles Foundation Discussion Papers*: **1164**.

[[PDF](#) (1.52 MB)]

165

M. & P. Laurent CALVET, Adlai FISHER, & M 1997. Large deviations and the distribution of price changes. *Cowles Foundation Discussion Papers*: **1165**.

[[PDF](#) (327 KB)]

166

M. & P. Adlai FISHER, Laurent CALVET, & M 1997. Multifractality of the Deutschmark/US Dollar exchange rates. *Cowles Foundation Discussion Papers*: **1166**.

[[PDF](#) (311 KB)]

P. Three fractal models in finance: discontinuity, concentration, risk. *Economic Notes (Siena)*: **26**, 1997, 139-178.

[[PDF](#) (10.30 MB)]

169

P. M 1999p. Renormalization and fixed points in finance, since 1962. *Statistical Physics 20, International IUPAP Conference (Paris, 1998)*. Edited by D. Iagolnitzer. *Physica*: **A263**, 1999, 477-487.

[[PDF](#) (4.50 MB)]

P. Fractal financial fluctuations: do they threaten sustainability? *Science for Survival and Sustainable Development*. Vatican City: Pontificia Academia Scientiarum, MM, 299-319.

[[PDF](#) (320.8 KB)]

172

M & P. M 2001a. Scaling in financial prices, I: Tails and dependence. *Quantitative Finance: 1*, 113-123.
[[PDF](#) (261 KB)]

• Reprint: *Beyond Efficiency and Equilibrium*. Edited by Doyne Farmer & John Geanakoplos, Oxford UK: The University Press, 2004.

173

M & P. M 2001b. Scaling in financial prices, II: Multifractals and the star equation. *Quantitative Finance: 1*, 124-130.
[[PDF](#) (108 KB)]

• Reprint: *Beyond Efficiency and Equilibrium*. Edited by Doyne Farmer & John Geanakoplos, Oxford UK: The University Press, 2004.

174

K, M & P. M 2001c. Scaling in financial prices, III: Cartoon Brownian motions in multifractal time. *Quantitative Finance: 1*, 427-440.
[[PDF](#) (224 KB)]

175

K, M & P. M 2001d. Scaling in financial prices, IV: Multifractal concentration. *Quantitative Finance: 1*, 641-649.
[[PDF](#) (205 KB)]

176

M & P. M 2001e. Stochastic volatility, power-laws and long memory. *Quantitative Finance: 1*, 558-559.

180

P. M 2003r. Heavy tails in finance for independent or multifractal price increments. *Handbook on Heavy Tailed Distributions in Finance*. Edited by Svetlozar T. Rachev (Handbooks in Finance: **30**, Senior Editor: William T. Ziemba): **1**, 1-34.
[[PDF](#) (295.1 KB)]

185

Parallel cartoons of fractal models in finance. *Annals of Finance: 1*, 179-192.
[[PDF](#) (230 KB)]

186

The inescapable need for fractal tools in finance. *Annals of Finance: 2*, 193-195.
[[PDF](#) (82 KB)]